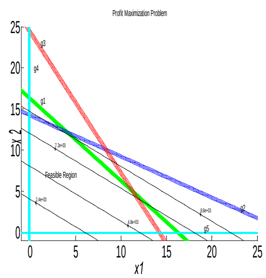
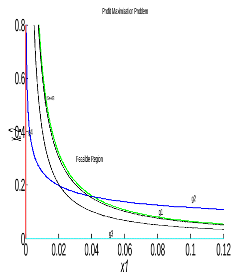
Lecture 05: Graphical Optimization Techniques





Outline

- Introduction
- Graphical Solution Process
  - Linear Problems
  - Nonlinear Problems

Profit Maximization Problem

STEP 1: PROJECT/PROBLEM DESCRIPTION A company manufactures two machines, A and B. Using available resources, either 28 A or 14 B can be manufactured daily. The sales department can sell up to 14 A machines or 24 B machines. The shipping facility can handle no more than 16 machines per day. The company makes a profit of $400 on each A machine and $600 on each B machine. How many A and B machines should the company manufacture every day to maximize its profit?

STEP 2: DATA AND INFORMATION COLLECTION Data and information are defined in the project statement.

STEP 3: DEFINITION OF DESIGN VARIABLES The following two design variables are identified in the problem statement:

x_1 = number of A machines manufactured each day
x_2 = number of B machines manufactured each day

Profit Maximization Problem

STEP 4: OPTIMIZATION CRITERION The objective is to maximize daily profit, which can be expressed in terms of design variables as

$$P = 400x_1 + 600x_2 \quad \text{(a)}$$

STEP 5: FORMULATION OF CONSTRAINTS

$$x_1 + x_2 \le 16 \text{ (shipping and handling constraint)} \quad \text{(b)}$$

$$\frac{x_1}{28} + \frac{x_2}{14} \le 1 \text{ (manufacturing constraint)} \quad \text{(c)}$$

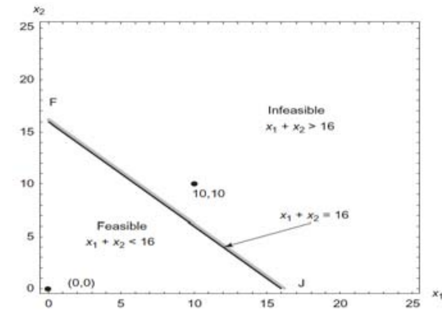
$$\frac{x_1}{14} + \frac{x_2}{24} \le 1 \text{ (limitation on sales department)} \quad \text{(d)}$$

$$x_1,\ x_2 \ge 0$$

Step-by-Step Graphical Solution Procedure

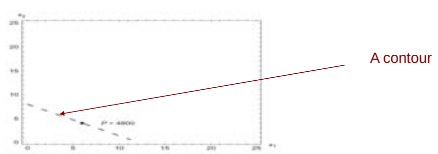
- Step 1: Coordinate System Set-up
- Step 2: Inequality Constraint Boundary Plot
- Step 3: Identification of the Feasible Region for an Inequality
- Step 4: Identification of the Feasible Region
- Step 5: Plotting of Objective Function Contours

Identification of Feasible Region for an Inequality

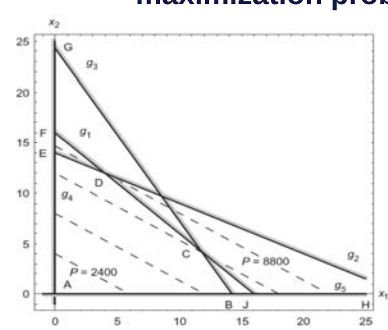


Objective Function Contour

- A contour is a curve on the graph that connects all points having the same objective function value.
- A collection of points on a contour is also called the level set.
- If the objective function is to be minimized, the contours are also called *isocost* curves.



Graphical solution to profit maximization problem



- Optimum point D = (4, 12); maximum profit, P = 8800.

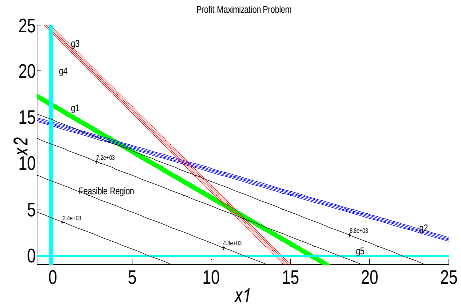
Use of MATLAB for Graphical Optimization

TABLE 3.1 MATLAB file for the profit maximization problem

m-file with explanatory comments

```
%Create a grid from -1 to 25 with an increment of 0.5 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.5:25.0,-1:0.5:25.0);
%Enter functions for the profit maximization problem
f=400*x1+600*x2;
g1=x1+x2-16;
g2=x1/28+x2/14-1;
g3=x1/14+x2/24-1;
g4=-x1;
g5=-x2;
%Initialization statements; these need not end with a semicolon
    cla reset
    axis auto    %Minimum and maximum values for axes are determined automatically
                 %Limits for x- and y-axes may also be specified with the command
                 %axis ([xmin xmax ymin ymax])
    xlabel('x1'),ylabel('x2') %Specifies labels for x- and y-axes
    title ('Profit Maximization Problem') %Displays a title for the problem
    hold on    %retains the current plot and axes properties for all subsequent plots
%Use the "contour" command to plot constraint and cost functions
    cv1=[0 .5];   %Specifies two contour values, 0 and .5
    const1=contour(x1,x2,g1,cv1,'k');   %Plots two specified contours of g1; k=black
                                          color
    clabel(const1)   %Automatically puts the contour value on the graph
    text(1,16,'g1')   %Writes g1 at the location (1, 16)
    cv2=[0 .03];
    const2=contour(x1,x2,g2,cv2,'k');
    clabel(const2)
    text(23,3,'g2')
    const3=contour(x1,x2,g3,cv2,'k');
    clabel(const3)
    text(1,23,'g3')
    cv3=[0 .5];
    const4=contour(x1,x2,g4,cv3,'k');
    clabel(const4)
    text(.25,20,'g4')
    const5=contour(x1,x2,g5,cv3,'k');
    clabel(const5)
    text(19,.5,'g5')
    text(1.5,7,'Feasible Region')
    fv=[2400, 4800, 7200, 8800];   %Defines 4 contours for the profit function
    fs=contour(x1,x2,f,fv,'k-');   %'k-' specifies black dashed lines for profit
                                    function contours
    clabel(fs)
    hold off   %Indicates end of this plotting sequence
               %Subsequent plots will appear in separate windows
```


Profit maximization problem with MATLAB

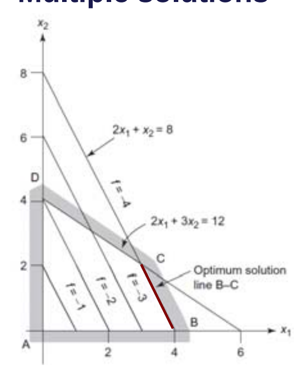


Design Problem with Multiple Solutions

- If a constraint is parallel to the cost function and the constraint is active at the optimum, there are multiple solutions to the problem.

Minimize	$f(\mathbf{x}) = -x_1 - 0.5x_2$	(a)
subject to	$2x_1 + 3x_2 \le 12,\ \ 2x_1 + x_2 \le 8,\ \ -x_1 \le 0,\ \ -x_2 \le 0$	(b)

Multiple solutions

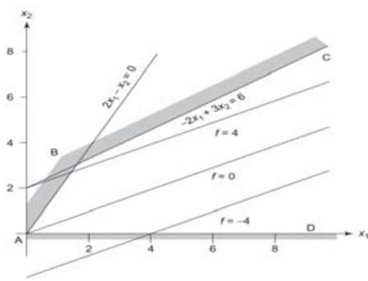


Problem with Unbounded Solutions

- This can happen when we forget a constraint or incorrectly formulate the problem.

Minimize	$f(\mathbf{x}) = -x_1 + 2x_2$	(c)
subject to	$-2x_1 + x_2 \le 0,\ \ -2x_1 + 3x_2 \le 6,\ \ -x_1 \le 0,\ \ -x_2 \le 0$	(d)

Unbounded Solutions



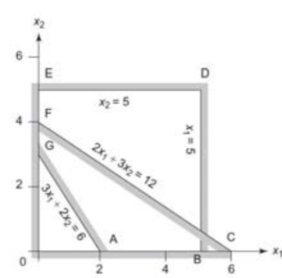
This problem is underconstrained.

Infeasible Problem

- This happens when there are conflicting requirements or inconsistent constraint equations.
- There may also be no solution when we put too many constraints on the system.

Minimize	$f(\mathbf{x}) = x_1 + 2x_2$	(e)
subject to	$3x_1 + 2x_2 \le 6,\ \ 2x_1 + 3x_2 \ge 12,\ \ x_1, x_2 \le 5,\ \ x_1, x_2 \ge 0$	(f)

Infeasible design optimization problem



Minimum-weight Tubular Column Design

STEP 1: PROJECT/PROBLEM DESCRIPTION Straight columns are used as structural elements in civil, mechanical, aerospace, agricultural, and automotive structures. Many such applications can be observed in daily life—for example, a street light pole, a traffic light post, a flag pole, a water tower support, a highway sign post, a power transmission pole. It is important to optimize the design of a straight column since it may be mass-produced. The objective of this project is to design a minimum-mass *tubular* column of length l supporting a load P without buckling or overstressing. The column is fixed at the base and free at the top, as shown in Figure 2.7. This type of structure is called a cantilever column.

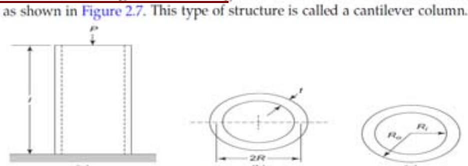


FIGURE 2.7 (a) Tubular column; (b) formulation 1; (c) formulation 2.

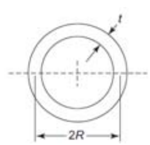
Minimum-weight Tubular Column Design

STEP 2: DATA AND INFORMATION COLLECTION The *buckling load* (also called the *critical load*) for a cantilever column is given as

$$P_{cr} = \frac{\pi^2 EI}{4l^2} \qquad (a)$$

- The column fails if the applied load exceeds the buckling load.
- The material stress σ for the column is defined as P/A, where A is the cross-sectional area of the column.
- The material allowable stress under the axial load is σ_a, and the material mass density is ρ (mass per unit volume).

Formulation 1 for Column Design



STEP 3: DEFINITION OF DESIGN VARIABLES For the first formulation, the following design variables are defined:

R = mean radius of the column
t = wall thickness

Assuming that the column wall is thin ($R \gg t$), the material cross-sectional area and moment of inertia are

$$A = 2\pi Rt; \quad I = \pi R^3 t \qquad (b)$$

Formulation 1 for Column Design

STEP 4: OPTIMIZATION CRITERION The total mass of the column to be minimized is given as

$$Mass = \rho(lA) = 2\rho l\pi Rt \qquad (c)$$

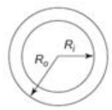
STEP 5: FORMULATION OF CONSTRAINTS

$$\frac{P}{2\pi Rt} \le \sigma_a \qquad (d)$$

$$P \le \frac{\pi^3 ER^3 t}{4l^2} \qquad (e)$$

$$R_{min} \le R \le R_{max}; \quad t_{min} \le t \le t_{max} \qquad (f)$$

Formulation 2 for Column Design



STEP 3: DEFINITION OF DESIGN VARIABLES Another formulation of the design problem is possible if the following design variables are defined:

R_o = outer radius of the column
R_i = inner radius of the column

In terms of these design variables, the cross-sectional area A and the moment of inertia I are

$$A = \pi(R_o^2 - R_i^2); \quad I = \frac{\pi}{4}(R_o^4 - R_i^4). \tag{g}$$

Formulation 2 for Column Design

STEP 4: OPTIMIZATION CRITERION Minimize the total mass of the column:

$$Mass = \rho(lA) = \pi\rho l(R_o^2 - R_i^2) \tag{h}$$

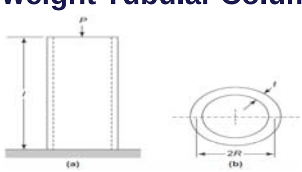
STEP 5: FORMULATION OF THE CONSTRAINTS

$$\frac{P}{\pi(R_o^2 - R_i^2)} \le \sigma_a \tag{i}$$

$$P \le \frac{\pi^3 E}{16 l^3}(R_o^4 - R_i^4) \tag{j}$$

$$R_{o\,min} \le R_o \le R_{o\,max}; \quad R_{i\,min} \le R_i \le R_{o\,max} \tag{k}$$

Graphical Solution for Minimum weight Tubular Column



- Data: P = 10 MN, E = 207 GPa, ρ = 7833 kg/m^3, l = 5.0 m, and σ = 5248 MPa.
- Find mean radius R (m) and thickness t (m) to minimize the mass function subject to constraints.

Tubular Column Design Problem Formulation

$$f(R,t) = 2\rho l\pi Rt = 2(7833)(5)\pi Rt = 2.4608 \times 10^5 Rt, \text{ kg} \tag{a}$$

s.t.

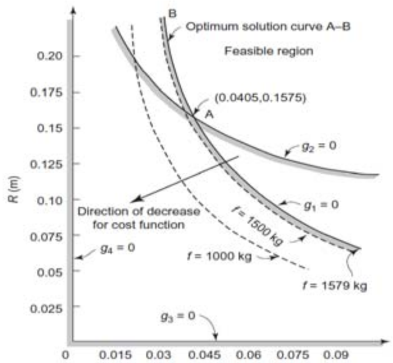
$$g_1(R,t) = \frac{P}{2\pi Rt} - \sigma_a = \frac{10 \times 10^6}{2\pi Rt} - 248 \times 10^6 \le 0 \quad \text{(stress constraint)} \tag{b}$$

$$g_2(R,t) = P - \frac{\pi^3 ER^3 t}{4l^2} = 10 \times 10^6 - \frac{\pi^3(207 \times 10^9)R^3 t}{4(5)(5)} \le 0 \quad \text{(buckling load constraint)} \tag{c}$$

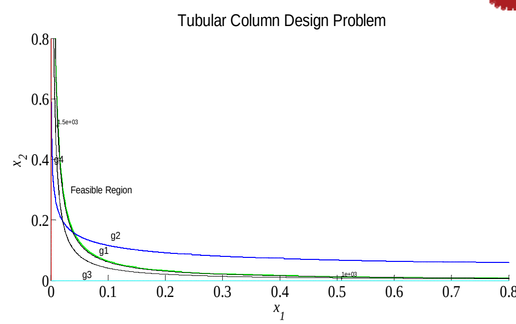
$$g_3(R,t) = -R \le 0 \tag{d}$$

$$g_4(R,t) = -t \le 0 \tag{e}$$

Graphical Solution



Graphical Solution



Assignment-2

- Arora Chapter 3:
  - 3.1, 3.6, 3.10, 3.28, 3.33.